Polymer 49 (2008) 4174–4179

Contents lists available at [ScienceDirect](www.sciencedirect.com/science/journal/00323861)

Polymer

journal homepage: www.elsevier.com/locate/polymer

Dimensions and viscosity behavior of polyelectrolyte brushes in aqueous sodium chloride. A polymacromonomer consisting of sodium poly(styrene sulfonate)

Eiji Kanemaru ^a, Ken Terao ^{a, "}, Yo Nakamura ^b, Takashi Norisuye ^a

a
Department of Macromolecular Science, Graduate School of Science, Osaka University, 1-1 Machikaneyama-cho, Toyonaka, Osaka 560-0043, Japan ^b Department of Polymer Chemistry, Graduate School of Engineering, Kyoto University, Katsura, Nishikyo-ku, Kyoto 615-8510, Japan

article info

Article history: Received 2 April 2008 Received in revised form 22 July 2008 Accepted 26 July 2008 Available online 31 July 2008

Keywords: Polyelectrolyte brush Light scattering Chain stiffness

ABSTRACT

Nine samples of a polymacromonomer consisting of sodium poly(styrene sulfonate) and having a sidechain polymerization degree of 15 are investigated by light scattering, small-angle X-ray scattering, and viscometry with 0.05 M aqueous NaCl at 25 \degree C as the solvent. The (total) weight-average molecular weight M_w ranges from 2.2 \times 10⁴ to 7.1 \times 10⁶. The radii of gyration, scattering functions, and intrinsic viscosities determined as functions of M_w are analyzed in terms of the cylindrical wormlike chain model. The estimated Kuhn segment length of about 120 nm is much larger than that (16 nm) for the polystyrene polymacromonomer with the equivalent side-chain length in toluene, a good solvent, while the chain thickness (5.5 nm) is comparable to or only slightly larger than that (5 nm) of the nonionic polymer. It is thus concluded that the electrostatic repulsion between side chains significantly stiffens the main chain of the polyelectrolyte brush.

- 2008 Elsevier Ltd. All rights reserved.

1. Introduction

Several decades ago, a few groups [\[1–4\]](#page-5-0) showed that the main chains of some comb-like polymers in the theta solvent are more extended than those of the corresponding linear chains due to the high segment density near the main chain. This chain extension becomes more pronounced with increasing side-chain length and shortening side-chain spacing on the main chain [\[5\],](#page-5-0) and comb polymers at the limit of the shortest spacing, i.e., so-called polymacromonomers [\[6,7\]](#page-5-0) or cylindrical brushes, behave as semiflexible chains with backbone stiffness higher for a longer side chain [\[8–](#page-5-0) [20\]](#page-5-0). Furthermore, their stiffness is higher in good solvents than in theta solvents [\[14–20\]](#page-5-0), indicating that interactions between or among side chains are also responsible for stiffening the comb backbone [\[21\].](#page-5-0) Thus it is intriguing and probably significant to investigate the effect of electrostatic repulsion on the backbone stiffness of a polyelectrolyte brush having an ionized group on each side-chain monomer unit, though the effective charge density may be much lower than what is expected for the corresponding linear polyelectrolyte [\[22\].](#page-5-0) As far as we know, no experimental work has yet been reported on the stiffness of an ionic polymacromonomer.

In the present work, we made light and small-angle X-ray scattering and viscosity measurements on a series of samples of a polymacromonomer consisting of sodium poly(styrene sulfonate) ([Fig. 1](#page-1-0)) in 0.05 M aqueous sodium chloride to estimate the backbone stiffness of the polyelectrolyte brush. We prepared these samples by sulfonation of the styrene units of polystyrene polymacromonomer samples [\[14\]](#page-5-0) with different main-chain lengths. The use of such polyelectrolyte samples has two advantages: (1) the dilute-solution behavior of each side chain, i.e., sodium poly- (styrene sulfonate), has extensively been studied [\[23–26\]](#page-5-0) and thus fairly well understood, and (2) the direct sulfonation should allow us to compare solution properties of the ionic and nonionic polymacromonomers having the exactly equivalent side-chain length. The radii of gyration, particle scattering functions, and intrinsic viscosities obtained for polyeletrolyte brush samples are analyzed below in terms of the wormlike chain [\[27\]](#page-5-0).

2. Experimental section

2.1. Samples

The previously investigated polymacromonomer F15 samples [\[14\]](#page-5-0) (F15-1, F15-2, ... F15-5, F15-11, F15-12 ... F15-14) having 15 styrene units on each side chain were sulfonated by Vink's method [\[28\]](#page-5-0). The typical procedure was as follows: a cyclohexane solution (70 cm³) of each F15 sample was mixed with 40 cm³ of concentrated sulfuric acid containing 7 g of P_2O_5 as a drying agent, followed by stirring for 2–3 h at 40 \degree C, and water was added to it at

^{*} Corresponding author. Tel.: $+81668505459$; fax: $+81668505461$. E-mail address: kterao@chem.sci.osaka-u.ac.jp (K. Terao).

^{0032-3861/\$ –} see front matter © 2008 Elsevier Ltd. All rights reserved. doi:10.1016/j.polymer.2008.07.047

Fig. 1. Repeating unit of a polymacromonomer consisting of sodium poly(styrene sulfonate). N and n denote the number of main-chain residues and that of side-chain residues, respectively.

 0° C. The water phase was extracted from the mixture, dialyzed against pure water for seven days, and mixed with a large amount of 0.5 M aqueous NaOH. The solution was then subjected to fractional precipitation (see Refs. [\[23,24\]](#page-5-0)) for the purpose of purifying the product. An appropriate middle fraction was passed through an ion exchanger column (after being dissolved in deionized water), neutralized, and freeze-dried in the manner employed for Na poly(styrene sulfonate) [\[24\].](#page-5-0)

The polymacromonomer samples (SF15) consisting of Na poly(styrene sulfonate) prepared and purified in this way were designated as SF15-1, SF15-2, ... SF15-9 in the order of descending molecular weight. The mass ratio of carbon to sulfate determined by elemental analysis indicated that these samples were almost fully sulfonated.

2.2. Light scattering

Scattering intensities for eight samples (SF15-1, SF15-2, ... SF15-8) of polymacromonomer SF15 in 0.05 M aqueous NaCl at 25° C were measured on a Fica-50 light scattering photometer with vertically polarized incident light of 436 or 546 nm wavelength (λ_0) . The experimental procedures were essentially the same as those described previously [\[13,20,26\]](#page-5-0). The weight-average molecular weight M_w , the second virial coefficient A_2 , and the z-average mean-square radius of gyration $\langle S^2 \rangle_z$ were determined by the square-root plots [\[29\]](#page-5-0) of $(Kc/R_\theta)^{1/2}$ vs $\sin^2(\theta/2)$ and vs c (the polymer mass concentration), where K and R_{θ} denote the optical constant and the reduced scattering intensity at scattering angle θ , respectively. Optical anisotropy effects on $M_{\sf w}$, A_2 , and $\langle S^2 \rangle_{\sf z}$ were negligible for the samples studied.

The specific refractive index increments $(\partial n/\partial c)_{\mu}$ at fixed chemical potentials μ of diffusible components and at 25 °C were measured for 0.05 M aqueous NaCl solutions of sample SF15-7 dialyzed against the solvent (see Ref. [\[30\]](#page-5-0) for the experimental details). The results of $(\partial n/\partial c)_{\mu}$ were 0.176 and 0.168 cm³ g⁻¹ at 436 and 546 nm, respectively.

2.3. Small-angle X-ray scattering (SAXS)

SAXS intensities I_{θ} at θ were measured for the three lowest molecular weight samples SF15-7–SF15-9 in 0.05 M aqueous NaCl at 25 °C on a Rigaku R-AXIS IV $++$ imaging plate at the BL40B2 beamline in SPring-8. Each test solution was filled in a 1.5 mm ϕ quartz capillary. The wavelength, the camera length, and the accumulation time were set to be 0.10 nm, 1580 mm, and 300 s, respectively. The scattering angle at each pixel on the imaging plate was determined from the Bragg reflection of powdery lead stearate. The excess scattering intensity ΔI_{θ} was obtained as the difference in I_{θ} between the solution and the solvent. Its concentration dependence and angular dependence were analyzed by use of the plots of

 $(c/\Delta I_\theta)^{1/2}$ vs c and $\ln(c/\Delta I_\theta)$ vs $\sin^2(\theta/2)$, respectively, and the particle scattering function $P(\theta)$ was obtained as a function of θ or k (the magnitude of the scattering vector) from the ratio of $(c/\Delta I_0)$ to $(c/$ ΔI_{θ}) both at infinite dilution.

2.4. Sedimentation equilibrium

Sedimentation equilibrium measurements were made on 0.05 M aqueous NaCl solutions of the lowest molar mass sample SF15-9 at 25° C in a Beckman Optima XL-1 ultracentrifuge to determine M_w , A_2 , and the z-average molecular weight M_z (see Ref. [\[31\]](#page-5-0) for the procedure of data analysis). A 12-mm double-sector cell was used and the rotor speed was chosen to be 20,000 rpm. The concentration profile was obtained from each interference pattern with a diode laser of $\lambda_0 = 675$ nm. The $(\partial n/\partial c)_\mu$ at this wavelength was estimated to be 0.163 $\rm cm^3\, g^{-1}$ from the data at 436 and 546 nm with the aid of the $(\partial n/\partial c)_{\mu}$ vs λ_0^{-2} plot. The density increment $(\partial \rho)$ ∂c _u at fixed μ was determined for SF15-7 in 0.05 M aqueous NaCl at 25 °C to be 0.374 using an Anton Paar DMA 5000 densitometer.

2.5. Viscometry

Zero-shear rate intrinsic viscosities $\lceil \eta \rceil$ in 0.05 and 0.005 M aqueous NaCl were determined at 25° C using a four-bulb lowshear viscometer of the Ubbelohde type for higher molecular weight samples SF15-1–SF15-3, and SF15-5 and capillary viscometers of the Ubbelohde type for the rest; some samples (SF15-4 at 0.05 M NaCl and SF15-3–SF15-5 at 0.005 M NaCl) were not studied because of their limited quantities. In evaluation of the relative viscosity η_r , the difference between the solution and solvent densities was taken into account.

3. Results

3.1. Dimensional properties

[Fig. 2](#page-2-0) illustrates the angular dependence of $(Kc/R₀)^{1/2}_{6–1}$, i.e., the infinite dilution value of (Kc/R_{θ}) ^{1/2}, for samples SF15-1, SF15-2, ... SF15-5 in 0.05 M aqueous NaCl. The intercepts and initial slopes of the indicated curves (the dashed lines) give M_w and $\langle S^2 \rangle_z$ for these samples.

Numerical results from light scattering measurements are summarized in [Table 1,](#page-2-0) along with those from sedimentation equilibrium. The M_w value for SF15-1 is about 40% smaller than the expected value, which is estimated to be 1.93 times as large as M_w of the precursor (i.e., polystyrene polymacromonomer F15). For the other SF15 samples, however, the difference between the expected and observed M_{w} s is less than $\pm 25\%$. Such a small difference is most likely due to the fractional precipitation mentioned in Section [2.](#page-0-0)

[Fig. 3](#page-2-0) shows the k^2 -dependence of $P(\theta)$ from SAXS in the form of the Guinier plot for SF15-7–SF15-9 in 0.05 M aqueous NaCl. The radii of gyration evaluated from the indicated straight lines (the dashed line for SF15-7) are presented in [Table 1.](#page-2-0)

The $\langle S^2 \rangle$ _z data for SF15 samples in the aqueous salt at 25 °C (the circles) are plotted against N_w (the weight-average number of main-chain residues) in [Fig. 4,](#page-2-0) along with our previous data [\[14,32,33\]](#page-5-0) for polystyrene polymacromonomer F15 in toluene at 15 °C (a good solvent, the triangles) and cyclohexane at 34.5 °C (a theta solvent, the squares). Here, N_w has been calculated from $M_w/$ M_0 with M_0 (the weight-average molecular weight of the macromonomer unit) = 3.18×10^3 for SF15 and 1.65×10^3 for F15 [\[14\].](#page-5-0) When compared at the same N_w, $\langle S^2 \rangle^{1/2}_2$ for SF15 is much larger than those for F15 in the two organic solvents for $N_w > 4 \times 10^2$, the difference being two to three times at $N_w = 2000$. As N_w decreases below 50, the dimensions of the ionic and nonionic polymacromonomers come close to one another. Since the latter

Fig. 2. Berry plots for indicated samples of the SF15 polymacromonomer in 0.05 M aqueous NaCl at 25 °C. (a) $\lambda_0 = 436$ nm (b) $\lambda_0 = 546$ nm.

polymer has a higher stiffness in the good solvent than in the theta solvent, the largest $\langle S^2 \rangle^{\frac{1}{2}}_z$ of the former polymer suggests the highest stiffness of the chain among the three systems.

3.2. Viscosity behavior

The fifth and sixth columns of Table 1 give data for $\left[\eta\right]$ and k' (Huggins' constant), respectively, where the bracketed values refer to 0.005 M aqueous NaCl at 25 $\,^{\circ}$ C. As expected for polyelectrolytes, $[\eta]$ is larger at this lower salt concentration than at 0.05 M. In connection with $[\eta]$ for low molecular weight samples, some remarks are pertinent here.

[Fig. 5\(](#page-3-0)a) and (b) shows the (ln η ^r)/c vs c plots for the four lowest molecular weight samples in 0.005 and 0.05 M aqueous NaCl, respectively. In the former figure, the curves remarkably bend down, revealing viscosity behavior typical of polyelectrolytes at low ionic strength. This remarkable bending does not allow the

Table 1

Molecular characteristics of polymacromonomer consisting of Na poly(styrene sulfonate) in 0.05 M aqueous NaCl at 25 $^{\circ}$ C

Sample	$10^{-5} M_{w}$	$\langle S^2 \rangle^{1/2}_z$ (nm)	$10^5 A_2$ (mol cm ³ g^{-2})	$[\eta]$ (cm ³ g ⁻¹)	k^{\prime}
SF15-1	71	104	4.4	170(230 ^b)	0.3(0.5 ^b)
SF15-2	68	97	3.7	117 $(191b)$	0.5(0.6 ^b)
SF15-3	35.5	54	4.8	59	0.6
SF15-4	29.5	57	9.2		
SF15-5	13.6	34.5	25	17.0	0.5
SF15-6	1.36		24	6.1	3
SF15-7	1.10	3.55	24	4.3	5
SF15-8	0.583	2.70	54	4.3	5
SF15-9	$0.220(0.26^a)$	2.23	180	6.0	3

^a 10^{–5} M_z.
^b Data in 0.005 M aqueous NaCl at 25 °C.

Fig. 3. Guinier plots for indicated SF15 samples in 0.05 M aqueous NaCl at 25 $^{\circ}$ C.

determination of $[\eta]$. In the latter figure, each curve still has a positive slope but less pronounced curvature, thus giving $[\eta]$ with moderate accuracy. These features of the viscosity plots are similar to what is known for Na poly(styrene sulfonate) of low molecular weight in the aqueous salts [\[25\].](#page-5-0)

[Fig. 6](#page-3-0) illustrates the N_w-dependence of $[\eta]M_0$ for the SF15 polyelectrolyte in 0.05 M aqueous NaCl (the unfilled circles) in comparison with that for the F15 polymer in good (toluene) and theta (cyclohexane) solvents [\[15\];](#page-5-0) the filled circles represent the data in 0.005 M aqueous NaCl. The value of $[\eta]$ for SF15 in 0.05 M aqueous NaCl is much larger than that for F15 at the same N_w above 10³. This is consistent with the $\langle S^2 \rangle$ data in Fig. 4, supporting the suggestion from $\langle S^2 \rangle_z$ that the backbone stiffness of the polyelectrolyte brush should be the highest among the three systems. As N_w lowers, $[\eta]$ for SF15 sharply decreases, becomes almost independent of N_w , and finally increases. This upswing of $[\eta]$ at low N_w is explained in the next section.

4. Discussion

4.1. Analysis of scattering data and molecular characteristics

We analyze the present scattering data for the SF15 polymacromonomer in 0.05 M aqueous NaCl using available theories

Fig. 4. N_w-dependence of $\langle S^2 \rangle^{\frac{1}{2}}_z$ for SF15 in 0.05 M aqueous NaCl at 25 °C (circles) and that for polystyrene polymacromonomer F15 in toluene at 15 \degree C (triangles, [\[14,33\]](#page-5-0)) and in cyclohexane at 34.5 °C (squares, [\[14,32\]](#page-5-0)).

Fig. 5. Concentration dependence of $(\ln \eta_r)/c$ for low molecular weight samples of SF15 in (a) 0.005 M and (b) 0.05 M aqueous NaCl at 25 $\,^{\circ}$ C. Open circles, SF15-6; filled circles, SF15-7; open triangles, SF15-8; filled triangles, SF15-9.

[\[34,35\]](#page-5-0) for the cylindrical wormlike chain. This model has successfully been applied to determine the molecular characteristics of nonionic polymacromonomers in both theta and good solvents [\[10–20\].](#page-5-0) Its mean-square radius of gyration $\langle S^2 \rangle$ is expressed by [\[34,36\]](#page-5-0).

$$
\left\langle S^{2}\right\rangle =\frac{L}{6\lambda}-\frac{1}{4\lambda^{2}}+\frac{1}{4\lambda^{3}L}-\frac{1}{8\lambda^{4}L^{2}}[1-exp(-2\lambda L)]+\frac{d^{2}}{8}
$$
 (1)

where L is the contour length, λ^{-1} is the Kuhn segment length, and d is the cylinder diameter. The first parameter is defined by the ratio of the molecular weight to the molar mass per unit contour length M_L , but we write it as [\[15\]](#page-5-0)

$$
L = \frac{M_{\rm w}}{M_{\rm L}} + \delta \tag{2}
$$

Fig. 6. N_w-dependence of [η]M₀ for SF15 in 0.05 M aqueous NaCl at 25 °C (open circles) and in 0.005 M aqueous NaCl at $25 °C$ (filled circles). Triangles and squares represent our previous data [\[15\]](#page-5-0) for F15 in toluene at 15 \degree C and in cyclohexane at 34.5 \degree C, respectively.

by taking into account the apparent contribution δ from side chains (near the main-chain ends) to L . Thus, Eq. (1) contains the four parameters $M_{\rm L}$, λ^{-1} , d, and δ .

We estimated them from the $\langle S^2 \rangle_z$ data by curve fitting to be $M_{\rm L}$ = 12,000 \pm 2000 nm⁻¹, λ^{-1} = 130 \pm 40 nm, d = 5.5 \pm 0.5 nm, and $\delta = 1.5 \pm 1.5$ nm. The first two parameters were substantially determined only by the data for $M_{\rm w}$ $>$ 10⁶, and thus the fitting to all the data points allowed the unique determination of the four parameters though the uncertainty of δ was large. Fig. 7(a) shows that the molecular weight dependence of $\langle S^2 \rangle^{1/2}$ for the polyelectrolyte brush is accurately described by Eq. (1) with M_{L} = 12,000 nm⁻¹, λ^{-1} = 130 nm, d = 5.5 nm, and δ = 1.5 nm. Importantly, this λ^{-1} value of 130 nm is about 8 and 14 times larger than those for the polystyrene polymacromonomer F15 in toluene and cyclohexane, respectively, demonstrating that sulfonation remarkably stiffens the backbone of the polymer. The estimated $M_{\rm I}$ yields 0.27 (\pm 0.05) nm for the contour length per repeat unit h $(equiv M_0/M_1)$. This h is close to the value (0.25 nm) expected for the trans-planar conformation of the main chain and also to those (0.26–0.27 nm) estimated from $\langle S^2 \rangle_z$ and $[\eta]$ for the F15 polymacromonomer in toluene and cyclohexane [\[14,15\]](#page-5-0). The diameter of the SF15 brush $(5.5 \pm 0.5 \text{ nm})$ is also comparable to or only slightly larger than those of F15 in toluene (5 nm) and cyclohexane (4.7 nm) [\[15,32,33\].](#page-5-0) We confirmed this diameter of SF15 by the cross-section plot, i.e., the $\ln[kP(\theta)]$ vs k^2 plot, for the present SAXS data, which gave essentially the same d of 6.0 nm. It should be noted that the d of 5.5–6.0 nm is considerably smaller than the fully stretched value of 7.5 nm for SF15 (=0.25 nm \times 15 \times 2).

In the above analysis of $\langle S^2 \rangle_z$, no excluded-volume effect was considered. According to our previous estimates [\[14,15\]](#page-5-0), this effect

Fig. 7. (a) Comparison between the experimental $\langle S^2 \rangle^{1/2}$ (circles) for SF15 in 0.05 M aqueous NaCl at 25 °C and the theoretical curve calculated from Eq. (1) with Eq. (2) for the cylindrical wormlike chain with $M_L = 12,000$ nm⁻¹, $\lambda^{-1} = 130$ nm, $d = 5.5$ nm, and $\delta = 1.5$ nm. (b) Comparison for [η]. The curve is calculated from Eq. [\(3\)](#page-4-0) with M_{L} = 14,000 nm⁻¹, λ^{-1} = 110 nm, d = 5.5 nm, and δ = 4.3 nm.

in toluene solutions of the F15 polymacromonomer becomes appreciable when the Kuhn segment number λ L exceeds 5–6. This critical value is comparable to the Kuhn segment number $5 (\pm 3)$ for our highest M_w sample. This coincidence appears to suggest negligible excluded-volume effects on $\langle S^2 \rangle_z$ for all the samples studied here, but a closer examination is necessary because excluded-volume interactions must be much stronger for the ionic SF15 polymer in 0.05 M aqueous NaCl than for the nonionic F15 polymer in toluene. In the framework of the Yamakawa–Stockmayer–Shimada theory [\[37–39\]](#page-5-0) for wormlike or helical-wormlike bead chains [\[39\]](#page-5-0), the radius expansion factor (=the ratio of perturbed to unperturbed $\langle S^2 \rangle^{1/2}$) is a universal function of the scaled excluded-volume parameter. For the two polymacromonomers with the same λL , this parameter differs only in $\lambda \beta_{2,b}$ (i.e., the binary cluster integral for the bead–bead interaction relative to λ^{-1}), and $\beta_{2,b}$ may be taken to equal the product of a constant C and the binary cluster integral β_2 for the side-chain units, i.e., for the monomeric units of Na poly(styrene sulfonate) or polystyrene; the constant C must be the same for the two polymacromonomers consisting of 15 side-chain residues and need not be specified here. Using $\beta_2 = 0.25$ nm³ for Na poly(styrene sulfonate) in the aqueous salt [\[25,26\]](#page-5-0) and $\beta_2 = 0.033$ nm³ for polystyrene in the organic solvent [\[39\],](#page-5-0) we get $\lambda\beta_{2,\text{b}}$ = 1.9 \times 10⁻³C and 2.1 \times 10⁻³C nm² for the SF15 and F15 systems, respectively. These $\lambda\beta_{2,b}$ s, quite close to each other, indicate that, when λL is the same, the two polymer-solvent systems undergo roughly the same degrees of chain expansion. Hence, we may conclude that our polyelectrolyte brush suffers no significant intramolecular excluded-volume effect in the M_w range concerned.

Fig. 8 compares the scattering functions (in the form of Kratky's plot) for samples SF15-7–SF15-9 in 0.05 M aqueous NaCl with the theoretical curves for the wormlike cylinders computed from Nakamura and Norisuye's theory [\[35\]](#page-5-0) with the set of parameters from $\langle S^2 \rangle_2^{1/2}$ ($M_L = 12{,}000$ nm⁻¹, $\lambda^{-1} = 130$ nm, $d = 5.5$ nm, and δ = 1.5 nm). The theory quantitatively reproduces the experimental curves for SF15-7 and SF15-8 having large peaks around $k = 0.5-$ 0.6 nm $^{-1}$, confirming the model parameters estimated from $\langle S^2 \rangle^{1/2}_z$.

Fig. 8. Kratky plots for (a) SF15-7, (b) SF15-8, and (c) SF15-9 in 0.05 M aqueous NaCl at 25 °C. Solid curves, theoretical values for the wormlike cylinder with M_L = 12,000 nm⁻¹, λ^{-1} = 130 nm, d = 5.5 nm, and δ = 1.5 nm.

The downward deviation of the curve for sample SF15-9 at $k > 1$ nm⁻¹ indicates that the cylinder model is too crude for the small number of side chains (only 7) at high scattering angles where X-ray sees the relatively local structure of the molecule. Similar behavior was observed for the corresponding F15 sample in cyclohexane [\[32\]](#page-5-0) and toluene [\[33\]](#page-5-0).

4.2. Analysis of viscosity data

The Yamakawa–Yoshizaki theory [\[40\]](#page-5-0) for $[\eta]$ of a wormlike spherocylinder may be written as

$$
[\eta] = \frac{f(\lambda L, \lambda d)}{\lambda^3 M} \tag{3}
$$

where M denotes the molecular weight of the polymer and $f(\lambda L, \lambda d)$ is a known function of λL and λd . The four parameters $M_{\rm L}$, λ^{-1} , d , and δ cannot uniquely be determined from the present [η] data. When *d* is assumed to be 5.5 nm obtained from $\langle S^2 \rangle^{\frac{1}{2}}$, the best fit to the experimental data was found for $M_{L} = 14,000$ nm⁻¹, $\lambda^{-1} = 110$ nm, and $\delta = 4.3$ nm by curve fitting, as shown in [Fig. 7](#page-3-0)(b). In actuality, however, the following parameter range led to substantially the same theoretical curves in the $M_{\rm w}$ range of our SF15 samples: ± 1000 nm⁻¹ for M_L, ± 30 nm for λ^{-1} , and ± 0.4 nm for δ . The M_L of 14,000 nm⁻¹ gives h a value of 0.23 nm, which does not differ much from 0.25 nm for the all trans conformation. The estimated stiffness is also close to that from $\langle S^2 \rangle^{1/2}_z$. Thus, we may conclude that the available theories for the wormlike cylinder consistently explain the data of $\langle S^2 \rangle_z$, $P(\theta)$, and [η] for the SF15 polyelectrolyte brush in 0.05 M aqueous NaCl.

We discuss the observed minimum in the log[η] vs log M_w relation [\(Fig. 6](#page-3-0) or [Fig. 7\)](#page-3-0) following the recent interpretation by Sugiyama et al. [\[20\]](#page-5-0) for a polystyrene polymacromonomer with 113 units in each side chain and confining ourselves to the rod limit of the wormlike spherocylinder. Taking into account the effect of δ (Eq. [\(2\)\)](#page-3-0), we have in this limit [\[41\],](#page-5-0)

$$
[\eta] \propto \frac{d^2 p^3}{p - (\delta/d)} F(p) \text{ (rod limit)}
$$
 (4)

where p is the axis ratio defined by L/d and $F(p)$ is a known function of p. As p decreases with both d and δ fixed, $p^3F(p)$ decreases while the factor $(p - \delta/d)^{-1}$ increases. As the result, $[\eta]$ has a broad minimum around $p = 2.3\delta/d$ (for $\delta/d = 0.78$), which corresponds to $N_{\rm w}$ $=$ 22 or $M_{\rm w}$ $=$ 7 \times 10⁴. Since the upswing of [η] with lowering $N_{\rm w}$ or M_w arises from the factor $(p - \delta/d)^{-1}$, the effect of δ on $[\eta]$ is significant at low molecular weights. In [Fig. 6,](#page-3-0) minima of $[\eta]s$ for the F15 polymacromonomer in the two organic solvents can hardly be seen (though there exist very shallow minima around $N_w = 15$). It can be shown by Eq. (4) that this viscosity behavior, seemingly contrasted to that of SF15, is due to the smaller δ/d (about 0.46) of the polystyrene polymacromonomer.

4.3. Main-chain stiffness

The backbone stiffness of the polyelectrolyte brush SF15 expressed in terms of the mean (120 nm) of the λ^{-1} values from $\langle S^2 \rangle^{1/2}_z$ and $[\eta]$ is 7.5 times higher than that (16 nm) of the polystyrene polymacromonomer F15 with the same side-chain degree of polymerization in toluene. This is due primarily to enhanced side chain–side chain interactions in the SF15 polymer. In other words, sulfonation of F15 greatly stiffens the polymacromonomer backbone, and electrostatic repulsions between charged groups of the resultant polyelectrolyte are responsible for the very high stiffness.

According to Nakamura and Norisuye's theory [\[21\],](#page-5-0) λ^{-1} is the sum of λ_b^{-1} (the increase of λ^{-1} arising from side chain–side chain

interactions) and λ_0^{-1} (the contribution of the intrinsic stiffness) and the former is given by λ_b^{-1} = (1/8 π)(n^2/h) β_2 , where n stands for the polymerization degree of each side chain; we note that the contribution from main chain – side chain interactions to λ_0^{-1} is proportional to *n* and ignored here. Using $\beta_2 = 0.25$ nm³ for Na poly(styrene sulfonate) in 0.05 M aqueous NaCl [25,26] and h = 0.25 nm, we obtain $\lambda^{-1}_{\rm b}$ = 36 nm, which is much smaller than the estimated λ^{-1} of 120 nm even though λ_0^{-1} must be added to λ_0^{-1} ; λ_0^{-1} for the polyelectrolyte brush with the bulky sulfonated groups should be larger than that (12 nm) often chosen for polystyrene polymacromonomers in toluene [20,21], but it is quite unlikely to compensate for the small $\lambda_{\rm D}^{-1}$. Thus, the present estimate of λ^{-1} may be taken to show that the Nakamura–Norisuye theory is inapplicable to the polyelectrolyte brush. This is attributable to the fact that the long-range nature of electrostatic interactions is not considered in the theory. The high backbone stiffness of polyelectrolyte brushes provides an interesting theoretical problem.

5. Conclusions

We determined radii of gyration, particle scattering functions, and intrinsic viscosities for a series of polymacromonomer samples SF15 consisting of sodium poly(styrene sulfonate) (with a side-chain polymerization degree of 15) in 0.05 M aqueous NaCl at 25 \degree C and analyzed them on the basis of theories [34,35,40] for the cylindrical wormlike chain. The theories consistently explain these data and yield mean values of 120, 0.25, and 5.5 nm for the Kuhn segment length, the contour length per main-chain residue, and the cylinder diameter, respectively. This Kuhn length is 7.5 times larger than that for the polystyrene polymacromonomer with the equivalent side-chain length in toluene, a good solvent. Electrostatic repulsions between charged groups of the polyelectrolyte brush are responsible for this backbone stiffening.

Acknowledgments

The synchrotron radiation experiments were performed at the BL40B2 in SPring-8 with the approval of the Japan Synchrotron Radiation Research Institute (JASRI) (Proposal No. 2007A1556).

References

- [1] Decker PD. Makromol Chem 1969;125:136–60.
- [2] Noda I, Horikawa T, Kato T, Fujimoto T, Nagasawa M. Macromolecules 1970;3:795–9.
- [3] Candau F, Rempp P. Eur Polym J 1972;8:757–68.
- [4] Strazielle C, Herz J. Eur Polym J 1977;13:223–33.
- [5] Terao K, Farmer BS, Nakamura Y, Iatrou H, Hong K, Mays JW. Macromolecules 2005;38:1447–50.
- [6] Tsukahara Y, Mizuno K, Segawa A, Yamashita Y. Macromolecules 1989; $22.1546 - 52$
- [7] Tsukahara Y, Tsutsumi K, Yamashita Y, Shimada S. Macromolecules 1990; 23:5201–8.
- [8] Barrales-Rienda JM, Galicia CR, Freire JJ, Horta A. Macromolecules 1983;16: $940 - 5$
- [9] Freire JJ, Barrales-Rienda JM, Galicia CR, Horta A. Macromolecules 1983;16: 946–50.
- [10] Wintermantel M, Schmidt M, Tsukahara Y, Kajiwara K, Kohjiya S. Macromol Rapid Commun 1994;15:279–84.
- [11] Wintermantel M, Gerle M, Fischer K, Schmidt M, Wataoka I, Urakawa H, et al. Macromolecules 1996;29:978–83.
- [12] Kawaguchi S, Akaike K, Zhang ZM, Matsumoto H, Ito K. Polym J 1998;30:1004–7.
- [13] Terao K, Takeo Y, Tazaki M, Nakamura Y, Norisuye T. Polym J 1999;31:193–8.
- [14] Terao K, Nakamura Y, Norisuye T. Macromolecules 1999;32:711–6.
- [15] Terao K, Hokajo T, Nakamura Y, Norisuye T. Macromolecules 1999;32:3690–4.
- [16] Hokajo T, Terao K, Nakamura Y, Norisuye T. Polym J 2001;33:481–5.
- [17] Fischer K, Schmidt M. Macromol Rapid Commun 2001;22:787–91.
- [18] Hokajo T, Hanaoka Y, Nakamura Y, Norisuye T. Polym J 2005;37:529–34.
- [19] Zhang B, Gröhn F, Pedersen JS, Fischer K, Schmidt M. Macromolecules 2006;39:8440–50.
- [20] Sugiyama M, Nakamura Y, Norisuye T. Polym J 2008;40:109–15.
- [21] Nakamura Y, Norisuye T. Polym J 2001;33:874–8.
- [22] Rühe J, Ballauff M, Biesalski M, Dziezok P, Gröhn F, Johannsmann D, et al. Adv Polym Sci 2004;165:79–150.
- [23] Takahashi A, Kato T, Nagasawa M. J Phys Chem 1967;71:2001–10.
- [24] Hirose E, Iwamoto Y, Norisuye T. Macromolecules 1999;32:8629–34.
- [25] Iwamoto Y, Hirose E, Norisuye T. Polym J 2000;32:428–34.
- [26] Yashiro J, Norisuye T. J Polym Sci Part B Polym Phys 2002;40:2728–35.
- Kratky O, Porod G. Recl Trav Chim Pays Bas 1949;68:1106-22.
- [28] Vink H. Macromol Chem 1981;182:279–81.
- [29] Berry GC. J Chem Phys 1966;44:4550-64.
- [30] Sato T, Norisuye T, Fujita H. Macromolecules 1983;16:185–9.
- [31] Norisuye T, Yanaki T, Fujita H. J Polym Sci Polym Phys Ed 1980;18:547–58.
- [32] Amitani K, Terao K, Nakamura Y, Norisuye T. Polym J 2005;37:324–31.
- [33] Nakamura Y, Sugiyama M, Amitani K, Norisuye T. Polym J 2007;39:1098–104.
- [34] Benoit H, Doty P. J Phys Chem 1953;57:958–63.
- [35] Nakamura Y, Norisuye T. J Polym Sci Part B Polym Phys 2004;42:1398–407.
- [36] Konishi T, Yoshizaki T, Saito T, Einaga Y, Yamakawa H. Macromolecules 1990;23:290–7.
- [37] Yamakawa H, Stockmayer WH. J Chem Phys 1972;57:2843–54.
- [38] Shimada J, Yamakawa H. J Chem Phys 1986;85:591–9.
- [39] Yamakawa H. Helical wormlike chains in polymer solutions. Berlin: Springer; 1997.
- [40] Yamakawa H, Yoshizaki T. Macromolecules 1980;13:633–43.
- [41] Yoshizaki T, Yamakawa H. J Chem Phys 1980;72:57–69.